Scientific Notation

When we use 10 as a factor 2 times, the product is 100.

$\quad 10^2 = 10 \times 10 = 100 \qquad\qquad$ second power of 10

When we use 10 as a factor 3 times, the product is 1000.

$\quad 10^3 = 10 \times 10 \times 10 = 1000 \qquad$ third power of 10.

When we use 10 as a factor 4 times, the product is 10,000.

$\quad 10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000 \qquad$ fourth power of 10.

From this, we can see that the number of zeros in each <u>product</u> equals the number of times 10 is used as a <u>factor</u>. The number is called a power of 10. Thus, the number

$\qquad\qquad\qquad$ 100,000,000

has **eight 0's** and must be the **eighth power of 10**. This is the product we get if 10 is used as a factor eight times!

Recall earlier that we've discussed that when multiplying any number by **powers of ten** that we move the **decimal to the right** the same number of times as **the number of zeros in the power of ten!**

$\qquad$ **Example :** $\quad 1.45 \times 1000 = 1{,}450$

Recall also that we've also discussed that when dividing any number by powers of ten that we **move the decimal to the left** the same number of times as **the number of zeros in the power of ten!**

$\qquad$ **Example :** $\quad 5.4792 \div 100 = 0.054792$

Because we now have a special way to write powers of 10 we can write the above two examples in a special way – it is called **<u>scientific notation</u>** .

$\qquad$ **Example :** $\quad 1.45 \times 10^3 = 1{,}450$ (since $10^3 = 1000$)

$\qquad$ **Example:** $\quad 5.4792 \times 10^{-2} = 0.054792 \quad$ (since $10^2 = 100$ and
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad [\,10^2\,]^{-1} = \dfrac{1}{100}$ which
$\qquad\qquad\qquad\qquad\qquad\qquad$ means divided by 100)

Writing a Number in Scientific Notation:

Step 1: Write the number so that it is a number ≥ 1 but < 10 (decimals can and will be used)

Step 2: *Multiply* this number by 10^x (x is a whole number) to tell your reader where the decimal point is really located. The **x** tells your reader how many places the decimal moved! (If the number was 1 or greater, then the **x** will be positive, telling your reader that you moved the decimal to the right to get back to the original number, otherwise the **x** will be negative telling the reader to move the decimal left to get back to the original number.)

> **Example :** Change 17,400 to scientific notation.
>   1) **Decimal**   1 7 4 0 0
>
>   2) **Multiply**   x 10
>
> **Example :** Write 0.00007200 in scientific notation
>
>   1) **Decimal**   0 0 0 0 7 2 0 0
>
>   2) **Multiply**   x 10
>
> **Example :**   Change each of the following to scientific notation
> a)   8,450   b)   104,050,001   c)   34
>
> d)   0.00902   e)   0.00007200   f)   0.92728

Note: When a number is written correctly in scientific notation, there is only one number to the left of the decimal. Scientific notation is always written as follows: a x 10^x, where a is a described above and x is an integer.

We also need to know how to change a number from scientific notation to <u>standard form</u>. This means that we **<u>write the number without exponents</u>**. This is very simple, we just use the definition of scientific notation to change it back – in other words, multiply the number by the factor of 10 indicated. Since **<u>multiplying</u>** a number by a factor of 10 simply **<u>moves the decimal to the right</u>** the number of times indicated by the # of zeros, that's what we do! If the exponent is negative, this indicates **<u>division</u>** by that factor of 10 so we would **<u>move the decimal to the left</u>** the number of times indicated by the exponent.

> **Example :**   Change 7.193×10^5 to standard form
>
>   1) Move Decimal to the Right _____ times.
>   2) Giving us the number …

Example : Change 6.259×10^{-3} to standard form.

          1) Move Decimal Left _____ times
          2) Giving us the number …

Example: Write each of the following to standard form.
a) 7.9301×10^{-3} b) 8.00001×10^{5}

c) 2.9050×10^{-5} d) 9.999×10^{6}

We can also use scientific notation to multiply and divide large numbers. This is really quite easy. Here is some explanation and how we can do it!

What happens if we wish to do the following problem,

$$7 \times 10^{2} \times 10^{3} = (7 \times 10^{2})(1 \times 10^{3})$$

We can think of 10^{2} and 10^{3} as **"decimal point movers."** The 10^{2} moves the decimal two places to the right and then the 10^{3} moves the decimal three more places to the right. When we are finished we have moved the decimal *five* places to the right. What happens in the equivalent expression with the whole numbers? Well, they are simply multiplied!

Steps for Multiplying with Scientific Notation:
Step 1: Multiply the whole numbers
Step 2: Add the exponents of the **"decimal point movers"**, the factors of 10.
Step 3: *Rewrite* in scientific notation where the number multiplied by the factor of 10 is
       ≥ 1 but < 10.

Before we begin practicing this concept, I want to practice a skill. I want to learn to write a number in **correct scientific notation**.

Steps for Writing in correct scientific notation
Step 1: Write the number in correct scientific notation
Step 2: Add the exponent of the new "number's factor of 10 and the one at the start.

     Example: Write in correct scientific notation.
     a) 14.4×10^{5} b) 105.4×10^{-3} c) 0.0005×10^{15}

     d) 0.098×10^{-4}

Example : Multiply and write the final answer in <u>correct scientific notation.</u>

a) $(3 \times 10^2)(2 \times 10^4)$ b) $(2 \times 10^{-2})(3 \times 10^6)$

c) $(1.2 \times 10^{-3})(12 \times 10^5)$ d) $(9 \times 10^7)(8 \times 10^{-3})$

Note: In part c) & d) once you multiply the numbers you have a number that is greater than 10 so it must be rewritten into correct scientific notation by thinking about the number that 14.4×10^{10} actually represents and changing that to scientific notation.

Steps for Dividing with Scientific Notation:
Step 1: Divide the whole numbers
Step 2: Subtract the exponents of the **"decimal point movers"** (numerator minus
Denominator exponents)
Step 3: *Rewrite* in scientific notation where the number multiplied by the factor of 10 is
≥ 1 but < 10.

Example: $\dfrac{(9 \times 10^5)}{(3 \times 10^2)} =$

Example: $\dfrac{(2.5 \times 10^7)}{(2.5 \times 10^5)} =$

Example: $\dfrac{(2 \times 10^{-2})}{(1.5 \times 10^5)} =$

Your Turn

1. Write the following in standard form.

a) 7.129×10^5 b) -6.02×10^{-3}

c) 8.0005×10^{-1} d) 2.10009×10^4

2. Write the following using correct scientific notation.

a) 0.0501   b) 72.0179

c) 8,000,000   d) 0.000008

3. Write in correct scientific notation.

a) 156×10^{12} b) 2897×10^{-13} c) 0.079×10^{14}

4. Multiply/Divide and write in correct scientific notation.

a) $(1.2 \times 10^2)(1.2 \times 10^5)$ b) $(2.5 \times 10^{-2})(2.5 \times 10^7)$

c) $\dfrac{(5.6 \times 10^2)}{(7 \times 10^5)}$ d) $\dfrac{(6.04 \times 10^{-2})}{(8 \times 10^{-3})}$

We can also use scientific notation to multiply/divide very large numbers easily. By putting the two factors into scientific notation and using the multiplication/division skills that we have just built to do the operation, it is much easier to keep track of all the zeros!

Example: Use scientific notation to multiply or divide (i.e. put the number into scientific notation and use your skills to multiply or divide them in scientific notation)

a) $10,000 \times 0.000027$ b) $2,500,000 \times 1,000,000$

c) 0.00012×0.00009 d) $\dfrac{10,500,000}{5,000}$

e) $\dfrac{0.0000005}{1,0000,000}$ f) $\dfrac{1.00005}{0.00003}$

Pythagorean Theorem
The Pythagorean Theorem deals with the length of the sides of a right triangle. The two sides that form the right angle are called the <u>legs</u> and are referred to as **a** and **b**. The side opposite the right angle is called the <u>hypotenuse</u> and is referred to as **c**. The Pythagorean Theorem gives us the capability of finding the length of one of the sides when the other two lengths are known. Solving the Pythagorean theorem for the missing side can do this. One of the legs of a right triangle can be found if you know the equation:

Pythagorean Theorem $\qquad\qquad a^2 + b^2 = c^2$

Solving the Pythagorean Theorem
Step 1: Substitute the values for the known sides into the equation
Note: **a** is a leg, **b** is a leg and **c** is the hypotenuse
Step 2: Square the values for the sides
Step 3: Solve using methods for solving quadratics or using principles of square roots (not covered by Blair)

Example: One leg of a right triangle is 7 ft. shorter than the other. The length of the hypotenuse is 13 ft. Find the lengths of the legs.

Example: The length of the hypotenuse is 13 m. One leg is two more than twice the other, find the lengths of the legs.